Dissipativity-Based Distributed Model Predictive Control with Low Rate Communication

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Distributed or networked model predictive control (MPC) can provide a computationally efficient approach that achieves high levels of performance for plantwide control, where the interactions between processes can be determined from the information exchanged among controllers. Distributed controllers may exchange information at a lower rate to reduce the communication burden. A dissipativity-based analysis is developed to study the effects of low communication rates on plantwide control performance and stability. A distributed dissipativity-based MPC design approach is also developed to guarantee the plantwide stability and minimum plantwide performance with low communication rates. These results are illustrated by a case study of a reactor-distillation column network. *AIChE J*, 61: 3288–3303, 2015
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Introduction

Model predictive control (MPC) has been one of the most popular advanced control techniques implemented in various industries,[1] as it can explicitly handle soft/hard constraints, provide high levels of performance, and explicitly adopt process models. Centralized MPC approaches, however, face difficulties when implemented on large-scale systems, (often consisting of more than half a dozen of process units,[2–4] due to their high complexity and heavy computational burden. Decentralized approaches divide a global objective into tractable local objectives, and assign them to local controllers. However, strong interactions between process units are often the characteristics of plantwide process systems.[5,6] In which case, decentralized controllers may achieve poor plantwide performance. Distributed model predictive control (DMPC) has received great attention in both of academia and industry.[7–9] In this approach, distributed controllers are designed to control a subsystem or a region of a large-scale system in a decentralized manner but with communications among controllers. Christofides and coworkers adopted a Lyapunov-based approach to ensure global stability for a process network with DMPC,[10] where a contractivity condition must be satisfied in order to guarantee plantwide stability. The system-wide stability and optimal performance also can be achieved by DMPC with distributed optimization.[9] The distributed approach deals with process interactions by exchanging information via a communication network or coordination such as agent negotiation,[11] price-driven negotiation,[12,13] sensitivity-based coordination,[14] and Dantzig–Wolfe decomposition.[15]

Dissipativity (or passivity, as a special case of dissipativity) is an input-output property of dynamical systems, which may be related to ℓ_2 gain and phase properties.[16–18] Dissipativity theory can be used as an effective tool for the quantitative stability and performance analysis of large-scale interconnected systems,[19] where the problem is decomposed into the analysis of the dissipativity of the subsystems and interconnection topology, (e.g., in Refs. 20 and 21). A dissipativity-based distributed control approach for a plantwide process systems was first developed by Xu and Bao,[22–24] where the closed-loop dissipativity constraint guarantees plantwide stability and minimum plantwide performance. Furthermore, a dissipativity-based DMPC was developed by adopting the dynamic supply rates in quadratic difference forms (QdFs) that fit the formulation of MPC naturally.[25]

When implementing a control system on a large-scale system, communication issues may be inevitable such as limited communication capacity, data losses, and irregular time delays, as studied in the literature (see Ref. 26 for more details). Naturally, communication issues should also be considered in DMPC design. To reduce communication requirement in spacecraft systems, Lavaei et al.[27] reformulated the distributed controller into a decentralized fashion. Maestre et al.[28] proposed an algorithm to resolve the problem caused by communication errors, where the controllers can operate in a decentralized way over a low-reliability communication network. El-Farra and coworkers[29–31] formulated a plantwide control system with a communication network as a hybrid system, where the maximum allowable update period can be determined without losing the exponential stability of the

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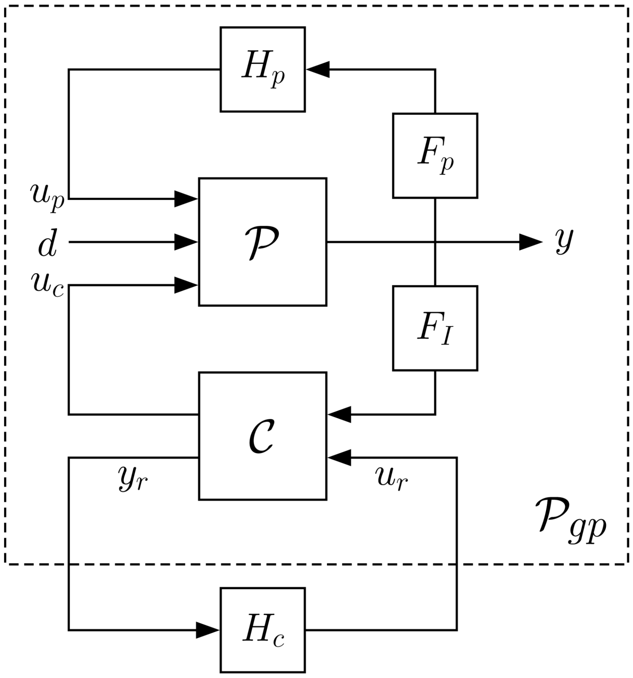


Figure 1. Structure of the plantwide system.

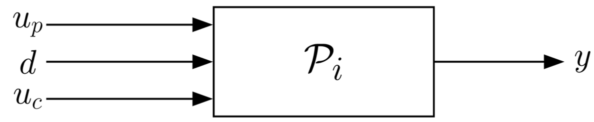


Figure 2. Partitioning of the *i*th process.

closed-loop plantwide system. Using a dissipativity-based approach, the effects of communication issues can be studied based on the input-output properties of process units and the topologies of the process and communication networks, such as in Refs. 32 and 33.

Many industrial control systems have limited communication bandwidths as deterministic networks are required for reliable communications. The bandwidth constraints can be worse with wireless communications,[34,35] which are often adopted by processes in remote areas such as mining and mineral processes. Therefore, it is important to study the feasibility and effect of reduced communication bandwidth for distributed MPC. We develop a new approach to the analysis of the effects of slow communication rates on plantwide stability and performance, wherein the distributed controllers communicate with one another at a rate slower than the sampling rate of individual subsystems. The proposed analysis is explicitly formulated in terms of the dissipativity properties of the plantwide system, by lifting it into a slower sampling rate (similar to Ref. 33), where the dissipativity of this lifted system can be explicitly handled using QdFs as supply rates. This analysis facilitates the design of dissipativity-based DMPC (as an extension of the approach initially developed in Ref. 25) with reduced communication rates.

The structure of this article is organized as follows. Distributed Control of Plantwide Systems section provides the formulation of distributed control of plantwide system with reduced-rate communication. In Dissipativity and Dynamical Supply Rate section, a brief introduction to dissipativity theory and dynamical supply rates is given. In Dissipativity-Based Analysis of the Effect of Communication Period on a Plantwide System section, we introduce a reduced-rate communication network and analyze the plantwide system with this network based on dissipativity. The proposed DMPC algorithm is then presented in Distributed Model Predictive Control section. These developments are illustrated by a case study in Illustrative Example section.

The following notations are used in this article. The time instant is denoted as k. Subscripts i and j denote the indexes of signal(s) of ith and jth unit. $\mathbb{R}^{\cdot\times\cdot}[\zeta,\eta]$ denotes the ring of two-variable polynomial matrices with real coefficients and arbitrary dimensions (where the important, dimensions are given). $\|\cdot\|_\ell$ denotes the ℓ-norm. Symbols $\mathbf{0}$ and I denote zero and identity matrices with appropriate dimensions, respectively. Symbols $\bar{\sigma}(A)$ and $\underline{\sigma}(A)$ denote the maximum and minimum singular values of the matrix A, respectively. The symbol $\mathbb{Z}^+$ denotes the set of nonnegative integers. The operator $\mathrm{diag}_{\tilde{\tau}}(Q)$ represents a block diagonal matrix $\mathrm{diag}(Q,\ldots,Q)$ with $\tilde{\tau}$ diagonal blocks of Q.

Distributed Control of Plantwide Systems

The structure of the plantwide system is depicted in Figure 1. The plantwide system consists of the collections of individuals processes and local controllers, which are introduced later. All physical flows are exchanged via the process topology H_p. The information between controllers exchanges via controller topology H_c.

The plantwide process $\mathcal{P}$ refers to the diagonal stack of individual processes (i.e., $\mathrm{diag}(\mathcal{P}_1,\ldots,\mathcal{P}_{n_p})$) in a chemical plant with n_p processes, where the ith process $\mathcal{P}_i$, depicted in Figure 2, is governed by an input-output model in Eq. 1

$$\mathcal{P}_i : y_i(k)=\sum_{j=1}^{n} A_{j_i}y_i(k-j)+\sum_{j=0}^{m} B_{1j_i}u_{p_i}(k-j)+B_{2j_i}u_{c_i}(k-j) +B_{3j_i}d_i(k-j) \tag{1}$$

In this model, we denote y_i, u_{p_i}, u_{c_i}, and d_i as its process output, process input, controlled input, and disturbance, respectively. The process input u_{p_i} is local feed physical flow, (such as materials and energy). The controlled input u_{c_i} is local optimal control actions and determined by the ith local controller. From the plantwide system point of view, those signals are stacked, such as $y=(y_1^T,\ldots,y_{n_p}^T)^T$.

Similarly, the stacked controller $\mathcal{C}$ refers to the collection of individual controllers without communication [i.e., $\mathcal{C}=\mathtt{diag}(\mathcal{C}_1,\ldots,\mathcal{C}_{n_p})$]. Figure 3 illustrates the partitioning of the ith controller with the local/remote controller input u_l/u_r and the local/remote controller output y_l/y_r, respectively. The pair of local signals, u_l/y_l, is the measurement from local process and local control action applied to local process, respectively. The pair of remote signals, y_r/u_r, is sent/received predicted trajectories that exchanges among controllers. The process network topology H_p and controller network topology H_c are constant matrices, which describe static interconnection structure in the plantwide process system. The filters, F_p and F_I, select interconnecting and measured outputs, respectively. In this work, they are considered as constant matrices with elements of 1 or 0. In such a structure, the physical process output y is selected and interconnected via topology H_p, that is, $(u_{1_p}^T\ldots u_{n_p}^T)^T=H_pF_p(y_1^T\ldots y_{n_p}^T)^T$ or $u_p=H_pF_py$. The ith controller receives selected

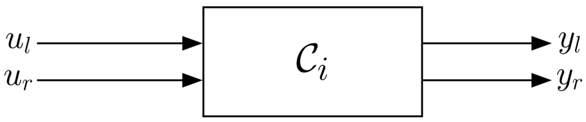


Figure 3. Partitioning of *i*th controller.

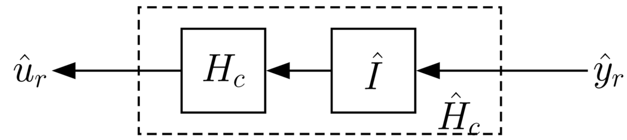


Figure 4. Communication network.

local process output, that is, $u_{l_i}=F_I y_i$. Each local controller sends its control policy to its local process, that is, $y_{l_i}=u_{c_i}$. The predicted trajectories are exchanged using a communication network with topology H_c. During communication, the remote input of ith controller, u_{r_i}, is the collections of the remote outputs from other controllers. Such remote input can be represented as the composite vector column of remote outputs y_{r_j} for some j, and therefore, $u_r=H_c y_r$ for the plantwide system. Those pairs of remote signals exchange trajectories via the communication network $\hat{H}_c$, which is modeled as a switched communication network shown in Figure 4. $\hat{H}_c$ consists of the controller topology H_c and a switching matrix $\hat{I}$. Normally, the unit communication rate may be designed as the same as process sampling rate. In this case, under the switching law $\hat{I}$, information only exchanges after $\tilde{\tau}$ communication period instead of unit communication period.

Dissipativity and Dynamical Supply Rate

The dissipativity property can be used to capture the dynamic features of a system.[18] It allows for large-scale systems to be analyzed in terms of their subsystems and interconnection topology by studying the dissipativity properties of these subsystems. Broadly speaking, a system is said to be dissipative if the change of "energy" (not necessarily physical energy) of the system is bounded by the net supply from the environment through the inputs and outputs. A discrete-time dynamic system is said to be dissipative with a supply rate, $s(y(k),u(k))$, if there exists a positive semidefinite storage function, $V(x(k))$, satisfying the dissipation inequality

$$V(x(k+1))-V(x(k)) \le s(y(k),u(k)) \qquad \forall k \ge 0 \tag{2}$$

where $x(k) \in \mathbb{R}^m$, $y(k) \in \mathbb{R}^n$, and $u(k) \in \mathbb{R}^q$ are state, output, and input at time instant k, respectively. Supply rates can be written in a quadratic form, which is (Q, S, R)-supply rate suggested by Hill and Moylan[36] as

$$s(y(k),u(k))=\begin{pmatrix} y(k) \\ u(k) \end{pmatrix}^T \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} y(k) \\ u(k) \end{pmatrix} \tag{3}$$

where Q, S, and R are matrices with appropriate dimensions. However, such supply rates only provide a conservative bound on the system dynamics. QdFs can be used to overcome this issue by including further information.[37] Denote a $\hat{n}_d$-degree extended signal space by $\hat{w}(k)=(\hat{y}^T(k),\hat{u}^T(k))^T$, where

$$\hat{y}(k)=(y^T(k),y^T(k+1),\ldots,y^T(k+\hat{n}_d))^T \tag{4a}$$

$$\hat{u}(k)=(u^T(k),u^T(k+1),\ldots,u^T(k+\hat{n}_d))^T \tag{4b}$$

A dynamic supply rate, $Q_\Phi(\hat{w}(k))=\hat{w}^T(k)\Phi\hat{w}(k)$, is defined in QdF induced by a polynomial matrix $\Phi(\zeta,\eta)$. Here, η denotes the forward shift operator of unit time, that is, $\eta w(k)=w(k+1)$. Similarly, ζ is defined as a forward shift operator of unit time on $w^T(k)$. They enjoy the property that $\zeta^T=\eta$. Using this notation, the quadratic form, $w^2(k)+2w^2(k+1)$, can be represented as

$$\begin{pmatrix} w(k) \\ w(k+1) \end{pmatrix}^T \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} w(k) \\ w(k+1) \end{pmatrix} = w^T(k)(1+2\zeta\eta)w(k) \tag{5}$$

QdF-dissipativity can be represented as $Q_\Phi=w^T(k)\Phi(\zeta,\eta)w(k)$, which is said to be induced by the two-variable polynomial matrix

$$\Phi(\zeta,\eta)=\sum_{i=0}^{\hat{n}_d}\sum_{j=0}^{\hat{n}_d}\zeta^i\eta^j\phi_{ij} \in \mathbb{R}^{\cdot\times\cdot}[\zeta,\eta] \tag{6}$$

Theorem 1. (Kojima and Takaba[37]). *A discrete linear time-invariant dynamical system is asymptotically stable, if there exists a symmetric nonnegative two-variable polynomial matrix $\psi(\zeta,\eta)$ satisfying the following inequality for all allowable output trajectories*

$$\Delta\psi(\zeta,\eta) < 0 \tag{7}$$

where $\Delta\psi(\zeta,\eta)=(\zeta\eta-1)\psi$ is the forward difference operator.

Theorem 1 is necessary and sufficient for the linear case, but only sufficient for the nonlinear case. In this article, all process units are described by input-output models as shown below

$$y(k)=\sum_{j=1}^{n}A_j y(k-j)+\sum_{j=0}^{m}B_j u(k-j) \tag{8}$$

Define the $\check{y}(k)$ as a backward lifted signal with appropriate degree $\check{n}$, that is, $\check{y}(k)=(y(k-\check{n})^T,\ldots,y(k)^T)^T$. The above model can be stacked over extended signal space to give the future prediction of process output trajectory $\hat{y}(k)$ as the function of the history of input $\check{u}(k)$ and output $\check{y}(k)$, and the future input $\hat{u}(k)$ (e.g., the MPC controller output)

$$\hat{y}(k)=\check{A}\check{y}(k)+\hat{B}\hat{u}(k)+\check{B}\check{u}(k) \tag{9}$$

where $\check{A}=(I-A)^{-1}\alpha$, $\check{B}=(I-A)^{-1}\beta$, $\hat{B}=(I-A)^{-1}B$ and A, B, α, β are Toeplitz matrices defined as follows

- A is upper triangular matrix with first row $(\mathbf{0},A_n,\ldots,A_1,\mathbf{0},\ldots,\mathbf{0})$
- B is lower triangular matrix with first column $(B_0,\ldots,B_m,\mathbf{0},\ldots,\mathbf{0})$
- α is upper triangular matrix with first row $(\mathbf{0},\ldots,\mathbf{0},A_n,\ldots,A_1)$
- β is upper triangular matrix with first row $(\mathbf{0},\ldots,\mathbf{0},B_m,\ldots,B_1)$

For example, A can be $\begin{pmatrix} \mathbf{0}_{2\times2} & \mathbf{1}_{2\times2} & I_{2\times2} & \mathbf{0}_{2\times2} \\ \mathbf{1}_{2\times2} & I_{2\times2} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} \\ I_{2\times2} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} \\ \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} \end{pmatrix}$, if $A_1=I_{2\times2}$ and $A_2=\mathbf{1}_{2\times2}=\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.

Proposition 1. (Zheng et al.[38]). *Consider a time-invariant dynamical system in Eq.* 8. *This system is said to be* *dissipative with respect to a supply rate Q_Φ induced by $\Phi(\zeta,\eta)=\begin{pmatrix}\phi_Q & \phi_S\\ \phi_S^T & \phi_R\end{pmatrix}$, and a storage function Q_Ψ induced by $\Psi(\zeta,\eta)=\begin{pmatrix}\psi_Q & \psi_S\\ \psi_S^T & \psi_R\end{pmatrix}\geq 0$, if and only if the following inequalities are satisfied*

$$\begin{pmatrix}\mathbb{T}_{11} & \mathbb{T}_{12} & \mathbb{T}_{13}\\ \mathbb{T}_{12}^T & \mathbb{T}_{22} & \mathbb{T}_{23}\\ \mathbb{T}_{13}^T & \mathbb{T}_{23}^T & \mathbb{T}_{33}\end{pmatrix}\geq 0 \tag{10a}$$

with

$$\mathbb{T}_{11}=\check{A}^T(\phi_Q-v_Q)\check{A} \tag{10b}$$

$$\mathbb{T}_{12}=\check{A}^T(\phi_Q-v_Q)\hat{B}+\check{A}^T(\phi_S-v_S) \tag{10c}$$

$$\mathbb{T}_{13}=\check{A}^T(\phi_Q-v_Q)\check{B} \tag{10d}$$

$$\mathbb{T}_{22}=\hat{B}^T(\phi_Q-v_Q)\hat{B}+\hat{B}^T(\phi_S-v_S)+(\phi_S-v_S)^T\hat{B}+(\phi_R-v_R) \tag{10e}$$

$$\mathbb{T}_{23}=\hat{B}^T(\phi_Q-v_Q)\check{B}+(\phi_S-v_S)^T\check{B} \tag{10f}$$

$$\mathbb{T}_{33}=\check{B}^T(\phi_Q-v_Q)\check{B} \tag{10g}$$

$$\begin{pmatrix}v_Q & v_S\\ v_S^T & v_R\end{pmatrix}=\Delta\Psi \tag{10h}$$

The lifting technique is used for the analysis of a time-varying system using extended signals. A reduced-rate communication can be formulated by imposing lengthened communication period, $\tilde{\tau}$, so that the communication only occurs every $\tilde{\tau}$ time units instead of every communication period. The dynamic behavior of this system can be captured by the signals after lifting, where the signal $\hat{y}(k)$ in this system lifts to $\left(\hat{y}^T(k),\ldots,\hat{y}^T(k+\tilde{\tau})\right)^T$. By using QdFs, the dissipativity of a dynamical system can be easily lifted into a slower sampling period. The following result allows for the dissipativity properties of a lifted system to be systemically determined from that of the original system and vice versa.

Lemma 1. (Tippett and Bao[39]). *Assume the supply rate of a dynamic system Σ, sampled at unit time, is induced by polynomial matrix $\Phi(\zeta,\eta)$ in Eq. 6. Let $\Sigma_{\tilde{\tau}}$ denotes the dynamic system lifted with a lengthened communication period $\tilde{\tau}$. Then, the QdF-supply rate of the lifted system is induced by the polynomial $\tilde{\Phi}(\zeta,\eta)$*

$$\tilde{\Phi}(\zeta,\eta)=\mathbb{L}_{\tilde{\tau}}^T(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Phi(\zeta,\eta))\mathbb{L}_{\tilde{\tau}}(\eta) \tag{11}$$

with the lifting operator $\mathbb{L}_{\tilde{\tau}}(\zeta)=(I\, I\zeta\,\cdots\, I\zeta^{\tilde{\tau}})$.

Example 1. *Consider the supply rate of a discrete-time system with unit sampling rate $\tau=1$, $Q_\Phi=\hat{w}^T(k)\begin{pmatrix}1&2&3\\2&4&5\\3&5&6\end{pmatrix}\hat{w}(k)$, induced by the polynomial matrix $\Phi=\begin{pmatrix}1 & \zeta & \zeta^2\end{pmatrix}\begin{pmatrix}1&2&3\\2&4&5\\3&5&6\end{pmatrix}\begin{pmatrix}1\\ \eta\\ \eta^2\end{pmatrix}$. This system is lifted with $\tilde{\tau}=1$. Its lifted supply rate, $Q_{\tilde{\Phi}}=\begin{pmatrix}\hat{w}(k)\\ \hat{w}(k+1)\end{pmatrix}^T\tilde{\Phi}\begin{pmatrix}\hat{w}(k)\\ \hat{w}(k+1)\end{pmatrix}$, is induced by the following polynomial matrix*

$$\tilde{\Phi}(\zeta,\eta)=\begin{pmatrix}I & \zeta I\end{pmatrix}\mathtt{diag}_2(\Phi)\begin{pmatrix}I\\ \eta I\end{pmatrix}=\Phi+\zeta\Phi\eta \tag{12}$$

The coefficient matrix, $\tilde{\Phi}$, can be easily calculated as

$$\begin{pmatrix}1&2&3&0\\2&4&5&0\\3&5&6&0\\0&0&0&0\end{pmatrix}+\begin{pmatrix}0&0&0&0\\0&1&2&3\\0&2&4&5\\0&4&5&6\end{pmatrix}=\begin{pmatrix}1&2&3&0\\2&5&7&3\\3&7&10&5\\0&4&5&6\end{pmatrix} \tag{13}$$

To facilitate the dissipativity-based analysis, we adopt the following concept of dissipative trajectory from our previous work.[25]

Definition 1. (Tippett and Bao[25]). *The ith controller $\mathcal{C}_i$ is said to trace a dissipative trajectory with respect to a supply rate, $Q_{\Phi_{c_i}}$, at all instants k, if the following inequality is satisfied within $t\in\mathbb{Z}^+$*

$$\mathcal{W}_{c_i}=\sum_{k=0}^{t}Q_{\Phi_{c_i}}(y(k),u(k))\geq 0 \tag{14}$$

Dissipativity-Based Analysis of the Effect of Communication Period on a Plantwide System

In the previous study,[25] the predicted process trajectories, (also known as remote control signals $\hat{y}_r(k)$), are exchanged among controllers at every k instant. Plantwide chemical plants, however, generally have slower plantwide dynamics than the dynamics of individual local processes.[40,41] Therefore, it is possible to use a slow communication rate but still achieve plantwide stability and sufficient plantwide performance.

In this section, we study the effects of reduced communication rate on a plantwide system based on dissipativity. The low rate communication can be modeled using a communication switch, which is introduced in Distributed Control of Plantwide Systems section. The lifting technique helps to analyze the effects of such time-varying system. With using QdF-dissipativity, the plantwide analysis of a lifted plantwide system can explicitly address the reduced communication rate.

Dissipativity formulation of *individual subsystems*

The process model in Eq. 1 can be lifted as Eq. 15, where $\check{A}$, $\check{B}_1$, $\check{B}_2$, $\check{B}_3$, $\hat{B}_1$, $\hat{B}_2$, $\hat{B}_3$ are similarly defined in Theorem 1:

$$\begin{aligned}\mathcal{P}_i:\hat{y}(k)=&\check{A}\check{y}(k)+\check{B}_1\check{u}_p(k)+\check{B}_2\check{u}_c(k)+\check{B}_3\check{d}(k)\\&+\hat{B}_1\hat{u}_p(k)+\hat{B}_2\hat{u}_c(k)+\hat{B}_3\hat{d}(k)\end{aligned} \tag{15}$$

Let $Q_{\Phi_i}(\hat{y},\hat{u})$ denote the QdF-supply rate of the ith process with respect to the extended signal space $(\hat{y}_i^T,\hat{u}_{p_i}^T,\hat{u}_{c_i}^T,\hat{d}_i^T)^T$, where this supply rate is induced by the polynomial matrix

$$\Phi_i(\zeta,\eta)=\begin{pmatrix}\mathcal{Q}_i(\zeta,\eta) & \mathcal{S}_i(\zeta,\eta)\\ \mathcal{S}_i^T(\zeta,\eta) & \mathcal{R}_i(\zeta,\eta)\end{pmatrix} \tag{16}$$

with

$$\mathcal{Q}_i=Q_i(\zeta,\eta) \tag{17}$$

$$\mathcal{S}_i=\left(S_{p_i}(\zeta,\eta),S_{c_i}(\zeta,\eta),S_{d_i}(\zeta,\eta)\right) \tag{18}$$

$$\mathcal{R}_i = \begin{pmatrix} R_{pp_i}(\zeta,\eta) & R_{pc_i}(\zeta,\eta) & R_{pd_i}(\zeta,\eta) \\ R_{pc_i}^T(\zeta,\eta) & R_{cc_i}(\zeta,\eta) & R_{cd_i}(\zeta,\eta) \\ R_{pd_i}^T(\zeta,\eta) & R_{cd_i}^T(\zeta,\eta) & R_{dd_i}(\zeta,\eta) \end{pmatrix} \tag{19}$$

Analogously, the QdF-supply rate of the ith controller $\mathcal{C}_i$ is induced by Φ_{c_i} with respect to the extended signal space $(\hat{y}_{l_i}^T, \hat{y}_{r_i}^T, \hat{u}_{l_i}^T, \hat{u}_{r_i}^T)^T$, where

$$\Phi_{c_i} = \begin{pmatrix} \mathcal{Q}_{c_i} & \mathcal{S}_{c_i} \\ \mathcal{S}_{c_i}^T & \mathcal{R}_{c_i} \end{pmatrix} \tag{20}$$

with

$$\mathcal{Q}_{c_i} = \begin{pmatrix} Q_{ll_i}(\zeta,\eta) & Q_{lr_i}(\zeta,\eta) \\ Q_{lr_i}^T(\zeta,\eta) & Q_{rr_i}(\zeta,\eta) \end{pmatrix} \tag{21}$$

$$\mathcal{S}_{c_i} = \begin{pmatrix} S_{ll_i}(\zeta,\eta) & S_{lr_i}(\zeta,\eta) \\ S_{rl_i}(\zeta,\eta) & S_{rr_i}(\zeta,\eta) \end{pmatrix} \tag{22}$$

$$\mathcal{R}_{c_i} = \begin{pmatrix} R_{ll_i}(\zeta,\eta) & R_{lr_i}(\zeta,\eta) \\ R_{lr_i}^T(\zeta,\eta) & R_{rr_i}(\zeta,\eta) \end{pmatrix} \tag{23}$$

Analysis of reduced-rate communication network

To aid the analysis of the effects of the controller communication network, the dissipativity of the process network with controllers without communication (stacked controllers) can be determined below

Lemma 2. *Consider system $\mathcal{P}_{gp}$, as shown in Figure 1, which consists of the plantwide process $\mathcal{P}$ and the stacked controller $\mathcal{C}$ with supply rates Q_Φ and Q_{Φ_c}, respectively. Denote Q_Ψ as the nonnegative storage function of $\mathcal{P}$, and H_p as the process network topology. Assume that the dissipativity of all processes are given in Proposition 1 and all controllers trace certain dissipative trajectories. If the input and output of $\mathcal{P}_{gp}$ satisify the following dissipative inequality for all $t \in \mathbb{Z}^+$*

$$\sum_{k=0}^{t} Q_M = \sum_{k=0}^{t} Q_\Phi + Q_{\Phi_c} \geq Q_\Psi \geq 0 \tag{24}$$

then, system $\mathcal{P}_{gp}$ is dissipative with respect to supply rate Q_M, is induced by

$$M = \begin{pmatrix} \mathbb{X}_{11} & \mathbb{X}_{12} & \mathbb{X}_{13} \\ \mathbb{X}_{12}^T & \mathbb{X}_{22} & \mathbb{X}_{23} \\ \mathbb{X}_{13}^T & \mathbb{X}_{23}^T & \mathbb{X}_{33} \end{pmatrix} \tag{25a}$$

where

$$\mathbb{X}_{11} = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12}^T & \Gamma_{22} & \Gamma_{23} \\ \Gamma_{13}^T & \Gamma_{23}^T & \Gamma_{33} \end{pmatrix} \tag{25b}$$

$$\mathbb{X}_{12} = (R_{lr}^T F_I \, S_{lr}^T \, S_{rr}^T)^T \tag{25c}$$

$$\mathbb{X}_{13} = (S_d^T + R_{pd}^T H_p F_p \, R_{cd}^T \, \mathbf{0})^T \tag{25d}$$

$$\mathbb{X}_{22} = R_{rr} \tag{25e}$$

$$\mathbb{X}_{23} = \mathbf{0} \tag{25f}$$

$$\mathbb{X}_{33} = R_{dd} \tag{25g}$$

$$\Gamma_{11} = Q + S_p H_p F_p + F_p^T H_p^T S_p^T + F_p^T H_p^T R_{pp} H_p F_p + F_I^T R_{ll} F_I \tag{25h}$$

$$\Gamma_{12} = S_c + F_p^T H_p^T R_{pc} + F_I^T S_{ll}^T \tag{25i}$$

$$\Gamma_{13} = F_I^T S_{lr} \tag{25j}$$

$$\Gamma_{22} = R_{cc} + Q_{ll} \tag{25k}$$

$$\Gamma_{23} = Q_{lr} \tag{25l}$$

$$\Gamma_{33} = Q_{rr} \tag{25m}$$

Proof. The dissipativity of $\mathcal{P}$ and $\mathcal{C}$ can be defined by the diagonally stacked dissipativity such as $Q = \mathtt{diag}(Q_1, \ldots, Q_i, \ldots, Q_{n_p})$. Furthermore, the controllers trace dissipative trajectories with the supply rate Q_{Φ_c}, and the plantwide process $\mathcal{P}$ is dissipative with respect to the supply rate Q_Φ. Therefore, $\sum Q_M = \sum Q_\Phi + Q_{\Phi_c} \geq Q_\Psi \geq 0$ for all k. Denote $w_{dp} = (\hat{y}^T, \hat{y}_l^T, \hat{y}_r^T, \hat{u}_r^T, \hat{d}^T)^T$. The supply rate of the plantwide process with the stacked controller shown in Figure 1 can rewritten as in a QdF in terms of w_{dp}, which is induced by the polynomial matrix M in Eq. 25. ■

Remark 1. It is worth noting that the above result includes the dissipativity of the remote signals. Furthermore, if controllers act in a decentralized manner, that is, $\hat{u}_r(k) = \hat{y}_r(k) = \mathbf{0}$, Eq. 25 reduces to $\begin{pmatrix} \mathbb{X}_{11} & \mathbb{X}_{13} \\ \mathbb{X}_{13}^T & \mathbb{X}_{33} \end{pmatrix}$.

The communication network $\hat{H}_c$ described in Distributed Control of Plantwide Systems Section can be represented as a static system after lifting into the period $\tilde{\tau}+1$ as

$$\hat{H}_c(\eta) = H_c \begin{pmatrix} P_0(\eta) I & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ P_{\tilde{\tau}}(\eta) I & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix} \tag{26}$$

In other words, the lifted input-output relationship of this communication network is governed by the following condition

$$\mathbb{L}_{\tilde{\tau}}(\eta) \hat{u}_r(k) = \hat{H}_c(\eta) \mathbb{L}_{\tilde{\tau}}(\eta) \hat{y}_r(k) \tag{27}$$

where $\mathbb{L}_{\tilde{\tau}}$ is lifting operator defined in Lemma 1.

Remark 2. The polynomial $P_\ell(\eta)$ in Eq. 27 describes the dynamics of the switch in the communication network at different instants. For example, the switch can act as zeroth-order hold, that is, $P_\ell(\eta) = I \quad \forall \ell \in \mathbb{Z}^+$ or first-order hold, that is, $P_\ell(\eta) = \eta^\ell I \quad \forall \ell \in \mathbb{Z}^+$.

The lifted dissipativity of this closed-loop can be obtained as follows.

Theorem 2. *Consider a lifted plantwide system which is represented as the closed loop system of $\tilde{\mathcal{P}}_{gp}$ and a reduced-rate communication network $\hat{H}_c$ (based on controller topology H_c in Eq. 26), as shown in Figure 5, where $\tilde{\mathcal{P}}_{gp}$ is the lifted system $\mathcal{P}_{gp}$ (as shown in Figure 1) with the communication period $\tilde{\tau}+1$. Consider conditions such that (1) the process model $\mathcal{P}$ (in Figure 1) is dissipative with a supply rate Q_ϕ with existing a storage function $Q_\Psi \geq 0$; (2) the stacked controller $\mathcal{C}$ traces nonnegative dissipative trajectories; (3) the communication network $\hat{H}_c$ is dissipative with respect to the supply rate $Q_{M_{\text{com}}}$ induced by $M_{\text{com}} = \begin{pmatrix} \mathcal{Q}_{\text{com}} & \mathcal{S}_{\text{com}} \\ \mathcal{S}_{\text{com}}^T & \mathcal{R}_{\text{com}} \end{pmatrix}$. For all external disturbances d and all* *instants $t \in \mathbb{Z}^+$, the plantwide system with processes and controller network is dissipative with respect to the supply rate $Q_{\tilde{M}}$ induced by*

$$\tilde{M} = \begin{pmatrix} \tilde{\mathbb{X}}_{11} & \tilde{\mathbb{X}}_{13} \\ \tilde{\mathbb{X}}_{13}^T & \tilde{\mathbb{X}}_{33} \end{pmatrix} \tag{28a}$$

where

$$\tilde{\mathbb{X}}_{11} = \begin{pmatrix} \tilde{\Gamma}_{11} & \tilde{\Gamma}_{12} & \tilde{\Gamma}_{13} \\ \tilde{\Gamma}_{12}^T & \tilde{\Gamma}_{22} & \tilde{\Gamma}_{23} \\ \tilde{\Gamma}_{13}^T & \tilde{\Gamma}_{23}^T & \tilde{\Gamma}_{33} \end{pmatrix} \tag{28b}$$

$$\tilde{\mathbb{X}}_{13} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{13})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{28c}$$

$$\tilde{\mathbb{X}}_{33} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{33})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{28d}$$

$$\tilde{\Gamma}_{11} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{11})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{28e}$$

$$\tilde{\Gamma}_{12} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{12})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{28f}$$

$$\tilde{\Gamma}_{13} = \mathbb{L}_{\tilde{\tau}}(\zeta)(\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{13}) + \mathtt{diag}_{\tilde{\tau}+1}(F_I^T R_{lr})\hat{H}_c)\mathbb{L}_{\tilde{\tau}}(\eta) \tag{28g}$$

$$\tilde{\Gamma}_{22} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{22})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{28h}$$

$$\tilde{\Gamma}_{23} = \mathbb{L}_{\tilde{\tau}}((\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{23}) + \mathtt{diag}_{\tilde{\tau}+1}(S_{lr})\hat{H}_c)\mathbb{L}_{\tilde{\tau}}(\eta) \tag{28i}$$

$$\begin{aligned} \tilde{\Gamma}_{33} = \mathbb{L}_{\tilde{\tau}}(\zeta)(\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{33} + \mathcal{R}_{\mathrm{com}}) + \hat{H}_c(\zeta) \\ \mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{22} + \mathcal{Q}_{\mathrm{com}})\hat{H}_c(\eta) + \mathtt{diag}_{\tilde{\tau}+1}(\mathcal{S}_{rr}^T + \mathcal{S}_{\mathrm{com}}^T)\hat{H}_c(\eta) \\ + \hat{H}_c(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\mathcal{S}_{rr} + \mathcal{S}_{\mathrm{com}}))\mathbb{L}_{\tilde{\tau}}(\eta) \end{aligned} \tag{28j}$$

Proof. The overall supply rate of the plantwide system is the linear combination of their lifted supply rates, which is induced by $\tilde{M} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(M + M_{\mathrm{com}})\mathbb{L}_{\tilde{\tau}}(\eta)$. According to Lemma 2, the closed-loop system satisfies the dissipative inequalities $\sum_{k=0}^{t} Q_{\tilde{M}} \geq Q_{\tilde{\Psi}} \geq 0$. The above result can be obtained using those definitions and formulation of network in Eqs. 26 and 27. ■

Together with Theorem 1, the following result provides the plantwide stability condition for the lifted plantwide system.

Theorem 3. *Consider a lifted plantwide system with $\mathcal{P}_{gp}$ and the controller network with conditions, as described in Theorem 2. This plantwide system from external disturbances $\hat{d}$ to plantwide output $\hat{y}_{pw} = (\hat{y}^T, \hat{y}_l^T, \hat{y}_r^T)^T$ is asymptotically stable if*

$$\tilde{\mathbb{X}}_{11} < 0 \tag{29}$$

Proof. To aid readability of this article, only a sketch of the proof is given. For vanishing disturbances, the lifted supply rate $Q_{\tilde{M}}$ becomes negative definite if $\tilde{\mathbb{X}}_{11} < 0$. Then, the dissipation inequality $Q_{\tilde{M}} \geq Q_{\nabla\tilde{\Psi}}$ implies $Q_{\nabla\tilde{\Psi}} < 0$. Together with Theorem 1 and $\Psi > 0$, this lifted plantwide system from external disturbances to plantwide output is asymptotically stable. ■

The following result is derived in the special case that the communication switch $\hat{I}$ acts as zero-order hold. In this result, the supply rate of the lifted plantwide system is explicitly formulated in term of the longer communication period $\tilde{\tau}+1$.

Corollary 1. *Consider a lifted plantwide system is represented as a closed loop of $\tilde{\mathcal{P}}_{gp}$ and the communication network $\hat{H}_c = \begin{pmatrix} H_c & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_c & 0 & \dots & 0 \end{pmatrix}$, as shown in Figure 5. All processes, controllers and the communication network follow the conditions as given in Theorem 2. Then, the lifted closed-loop system is dissipative with respect to the supply rate $Q_{\tilde{M}'}$ induced by*

$$\tilde{M}' = \begin{pmatrix} \tilde{\mathbb{X}}_{11} & \tilde{\mathbb{X}}_{13} \\ \tilde{\mathbb{X}}_{13}^T & \tilde{\mathbb{X}}_{33} \end{pmatrix} \tag{30}$$

where

$$\tilde{\mathbb{X}}_{11} = \begin{pmatrix} \tilde{\Gamma}_{11} & \tilde{\Gamma}_{12} & \tilde{\Gamma}_{13} \\ \tilde{\Gamma}_{12}^T & \tilde{\Gamma}_{22} & \tilde{\Gamma}_{23} \\ \tilde{\Gamma}_{13}^T & \tilde{\Gamma}_{23}^T & \tilde{\Gamma}_{33} \end{pmatrix} \tag{31}$$

$$\tilde{\mathbb{X}}_{13} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{13})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{32}$$

$$\tilde{\mathbb{X}}_{33} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\mathbb{X}_{33})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{33}$$

$$\tilde{\Gamma}_{11} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{11})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{34}$$

$$\tilde{\Gamma}_{12} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathtt{diag}_{\tilde{\tau}+1}(\Gamma_{12})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{35}$$

$$\tilde{\Gamma}_{13} = \mathbb{L}_{\tilde{\tau}}(\zeta)\begin{pmatrix} \Gamma_{13} + (\tilde{\tau}+1)F_I^T R_{lr} H_c & \mathbf{0} \\ \mathbf{0} & \mathtt{diag}_{\tilde{\tau}}(\Gamma_{13})) \end{pmatrix}\mathbb{L}_{\tilde{\tau}}(\eta) \tag{36}$$

$$\tilde{\Gamma}_{22} = \mathbb{L}_{\tilde{\tau}}(\zeta)\mathrm{diag}_{\tilde{\tau}+1}(\Gamma_{22})\mathbb{L}_{\tilde{\tau}}(\eta) \tag{37}$$

$$\tilde{\Gamma}_{23} = \mathbb{L}_{\tilde{\tau}}(\zeta)\begin{pmatrix} \Gamma_{23} + (\tilde{\tau}+1)S_{lr} H_c & \mathbf{0} \\ \mathbf{0} & \mathtt{diag}_{\tilde{\tau}}(\Gamma_{23})) \end{pmatrix}\mathbb{L}_{\tilde{\tau}}(\eta) \tag{38}$$

$$\tilde{\Gamma}_{33} = \mathbb{L}_{\tilde{\tau}}(\zeta)\begin{pmatrix} \Gamma_{33} + \mathcal{R}_{\mathrm{com}} + (\tilde{\tau}+1)(\mathcal{Q}_{\mathrm{com}} + H_c^T(\mathbb{X}_{22} + \mathcal{Q}_{\mathrm{com}})H_c + H_c^T(\mathcal{S}_{rr} + \mathcal{S}_{\mathrm{com}}) + (\mathcal{S}_{rr}^T + \mathcal{S}_{\mathrm{com}}^T)H_c) & \mathbf{0} \\ \mathbf{0} & \mathtt{diag}_{\tilde{\tau}}(\Gamma_{33} + \mathcal{R}_{\mathrm{com}})) \end{pmatrix}\mathbb{L}_{\tilde{\tau}}(\eta) \tag{39}$$

Proof. The proof is similar to Theorem 2, but using the communication network $\hat{H}_c = \begin{pmatrix} H_c & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ H_c & \mathbf{0} & \dots & \mathbf{0} \end{pmatrix}$. Using this switching network, the lifted signal

$$(\hat{u}_r^T(k), \dots, \hat{u}_r^T(k+\tilde{\tau}))^T = (H_c\hat{y}_r^T(k), \dots, H_c\hat{y}_r^T(k))^T \tag{40}$$

Then, the induced polynomial matrix $\tilde{M}$ becomes $\tilde{M}'$. ■

Remark 3. Theorem 2 and Corollary 1 can be interpreted by employing dynamic communication error gain from $\hat{y}_r$ *to* $\Delta\hat{y}_r$, which is the sensitivity of control system to the changes

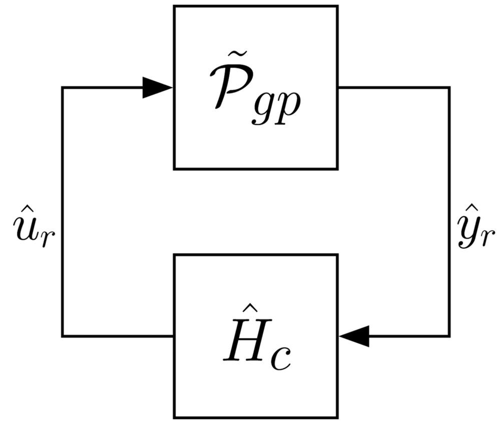


Figure 5. Plantwide system with communication network.

in exchanged information. For the purpose of illustration, the plantwide system may be virtually partitioned as shown in Figure 6. We assume that $\tilde{\mathcal{P}}_{gp}$ is stable with regular communication, where the communication network is the series connection of the controller topology H_c and the lifted identity matrix $\tilde{I}=\operatorname{diag}_{\tilde{\tau}+1}(I)$ (i.e., DMPC controllers communicate during every communication period). The lower block in the dashed box represents the difference between regular communication and reduced-rate communication, which is formulated as

$$\Delta\hat{I}=\hat{I}-\tilde{I}=\begin{pmatrix} \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ I & -I & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ I & \mathbf{0} & \dots & -I \end{pmatrix} \tag{41}$$

It has a maximum singular value $\delta_{\max}$ *of* $\sqrt{\tilde{\tau}+1}$. The value of $\delta_{\max}$ implies that longer communication periods (as described by $\tilde{\tau}$) yields larger upper bounds on the dynamic communication error gain. Using small gain arguments, this in turn requires the gain of the upper loop in Figure 6 (i.e., the sensitivity of the control system to communication) to be smaller to maintain stability with reduced-rate communication.

Remark 4. If an alternative higher order switch is used, the corresponding $\Delta\hat{I}$ can be formulated as $\hat{I}-\tilde{I}+\Delta$, where Δ is the difference between the higher order hold and zero-order hold. If Δ is treated as uncertainty, the use of a higher (than zero) order hold may lead to a larger maximum singular value of the new $\Delta\hat{I}$, which implies a (possible) lower level of worse case achievable plantwide performance. This is consistent with the result reported in Refs. 42 and 43.

The following result guarantees the minimum plantwide performance of the lifted plantwide system with the communication period $\tilde{\tau}+1$.

Theorem 4. *Consider a lifted plantwide system with conditions, as described in Theorem 2. If the supply rate of the lifted plantwide system, from external disturbances $\hat{d}$ to the plantwide outputs $\hat{y}_{\text{pw}}=(\hat{y}^T, \hat{y}_l^T, \hat{y}_r^T)^T$, is induced by $\tilde{M}=$*

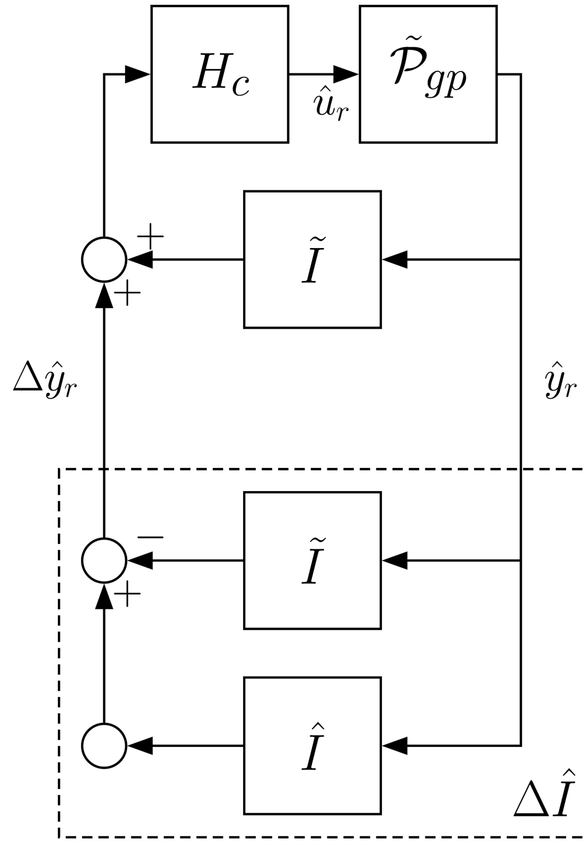


Figure 6. Virtual partitioning of the lifted plantwide system with reduced-rate communication.

$\begin{pmatrix} \tilde{\mathbb{X}}_{11} & \tilde{\mathbb{X}}_{13} \\ \tilde{\mathbb{X}}_{13}^T & \tilde{\mathbb{X}}_{33} \end{pmatrix}$ *with* $\tilde{\mathbb{X}}_{11}<0$, *then the minimum plantwide performance level*

$$||\mathcal{W}\tilde{\hat{y}}_{\text{pw}}||_2 \leq ||\tilde{\hat{d}}||_2 \tag{42}$$

is guaranteed with

$$\mathcal{W}(\eta)=\frac{1}{p(\eta)}\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}} \tag{43}$$

where $\underline{\sigma}(\mathcal{W}(j\omega)) \geq \frac{1}{\gamma}\ \forall\omega \in [0, 2\pi]$ *and a scalar polynomial p* (η) *such that its coefficient column vector, p, satisfies* $p^T p \geq max\,(\bar{\sigma}(\tilde{\mathbb{X}}_{33}+\tilde{\mathbb{X}}_{13}^T\hat{\tilde{\mathbb{X}}}_{11}\tilde{\mathbb{X}}_{13}), \bar{\sigma}(\tilde{\mathbb{X}}_{13}^T\hat{\tilde{\mathbb{X}}}_{11}\tilde{\mathbb{X}}_{13}))$.

Proof. See Theorem 4 section of Appendix A. ■

Remark 5. The proposed approach can be used to determine if the plantwide stability and a certain level of closed-loop performance can be achieved for a given communication rate, process models, and controller communication network topology. While it will be difficult to determine the minimum allowable communication rate through convex optimization, it can be obtained by iteratively checking the feasibility of dissipativity conditions that imply the plantwide stability and required plantwide performance.

Distributed Model Predictive Control

In this section, we present an approach to DMPC which allows for less frequent information exchange among controllers. The approach includes both offline and online procedures. The offline procedure is the determination of the dissipativity conditions for each individual MPC local controllers that ensure plantwide stability and performance. Based on Proposition 1, Theorems 3 and 4, this can be formulated into an linear matrix inequality (LMI) problem, which can be solved efficiently (e.g., using the approach in Ref. 44). It also serves as a feasibility test of the proposed approach for the given communication rate, which hinges on the existence of the solution to the above problem (i.e., the dissipativity conditions for all controllers). Each controller implements an online MPC algorithm (with a small computational burden) which optimizes a local cost function specified by the user subjected to a dissipative constraint that ensures plantwide stability and performance.

Constraints of controller dissipativity and their feasibility

The first step is to determine the dissipatvity conditions that each local controller needs to satisfy to ensure plantwide stability and performance. There are two issues here: the dissipativity property of any given process is not unique. Therefore, the most "suitable" supply rates of process units need to be determined together with the required dissipativity conditions for the local controllers so that they are "compatible" with each other. The other problem is that the dissipativity condition for a local controller will be implemented as an additional constraint of the MPC algorithm. As such, it is important to ensure the MPC problem is feasible with such a constraint (a recursive feasibility issue). In the following results, we formulate the recursive dissipative trajectory into an LMI condition, which can be determined offline and implemented online. The computational complexity of the online implementation is very low, as each controller only needs to solve its local optimization problem subject to one dissipativity condition. Also, a sufficient condition is provided to ensure recursive feasibility.

At any time k, the supply rate of the ith controller $Q_{\Phi_{c_i}}(\zeta,\eta)$ specifies the relationship between its current inputs and outputs as well as predicted future inputs and outputs, which is described below

Proposition 2. *Assume that the model of local system is given as Eq.* 8 *and the supply rate of ith controller required for plantwide stability and performance is induced by* $\Phi_{c_i}=\begin{pmatrix}\mathcal{Q}_{c_i} & \mathcal{S}_{c_i}\\ \mathcal{S}_{c_i}^T & \mathcal{R}_{c_i}\end{pmatrix}$, *which has been determined offline. This controller traces the dissipative trajectory specified by* Φ_{c_i} *at any point in time* $[k,k+\tilde{N})$, *if the following LMI is satisfied*

$$\begin{pmatrix} -\chi_i^{-1} & \hat{u}_{c_i}(k)\\ \hat{u}_{c_i}^T(k) & \Omega_i+W_{c_i}(k-\tilde{N}-1)\end{pmatrix}\geq 0 \tag{44}$$

where

$$\chi_i=(Q_{lr_i}+S_{ll_i})\hat{B}_{2_i}+\hat{B}_{2_i}^T(Q_{lr_i}+S_{ll_i})^T+\hat{B}_{2_i}^T\mathbb{F}\hat{B}_{2_i} \tag{45a}$$

$$\Omega_i=\begin{pmatrix}\Omega_{11_i} & \Omega_{12_i}\\ \Omega_{12_i}^T & \Omega_{22_i}\end{pmatrix} \tag{45b}$$

$$\Omega_{11_i}=\begin{pmatrix}\mathbf{0} & (Q_{lr_i}+S_{ll_i})\hat{B}_{1_i}+S_{lr_i}+\hat{B}_{2_i}^T\mathbb{F}\hat{B}_{1_i}+\hat{B}_{2_i}R_{lr_i}\\ * & \hat{B}_{1_i}^T\mathbb{F}\hat{B}_{1_i}+\hat{B}_{1_i}^T(S_{rr_i}^T+R_{lr_i})+(S_{rr_i}^T+R_{lr_i})^T\hat{B}_{1_i}+R_{rr_i}\end{pmatrix} \tag{45c}$$

$$\Omega_{12_i}=\begin{pmatrix}(Q_{lr_i}+S_{ll_i}+\hat{B}_{2_i}^T\mathbb{F})\check{A}_i, & (Q_{lr_i}+S_{ll_i}+\hat{B}_{2_i}^T\mathbb{F})\check{B}_{2_i}, & (Q_{lr_i}+S_{ll_i}+\hat{B}_{2_i}^T\mathbb{F})\check{B}_{1_i}\\ (\check{B}_{2_i}^T\mathbb{F}+S_{rr_i}^T+R_{lr_i})\check{A}_i, & (\check{B}_{2_i}^T\mathbb{F}+S_{rr_i}^T+R_{lr_i})\check{B}_{2_i}, & (\check{B}_{2_i}^T\mathbb{F}+S_{rr_i}^T+R_{lr_i})\check{B}_{1_i}\end{pmatrix} \tag{45d}$$

$$\Omega_{22_i}=\begin{pmatrix}\check{A}_i^T\mathbb{F}\check{A}_i & \check{A}_i^T\mathbb{F}\check{B}_{2_i} & \check{A}_i^T\mathbb{F}\check{B}_{1_i}\\ * & \check{B}_{2_i}^T\mathbb{F}\check{B}_{2_i} & \check{B}_{2_i}^T\mathbb{F}\check{B}_{1_i}\\ * & * & \check{B}_{1_i}^T\mathbb{F}\check{B}_{1_i}\end{pmatrix} \tag{45e}$$

$$\mathbb{F}=Q_{rr_i}+S_{rl_i}+S_{rl_i}^T+R_{ll_i} \tag{45f}$$

Proof. See Proposition 3 section of Appendix A. ■

The above result allows for the formulation of the online dissipative trajectory in a convex manner, as given in the following problem

Problem 1. *(Offline). Consider a lifted* n_p*-process plantwide system with the communication period* $\tilde{\tau}+1$ *as described in Theorem 2, where the QdF-supply rates of ith process, ith controller and communication network are* Q_{Φ_i}, $Q_{\Phi_{c_i}}$, *and* $Q_{M_{com}}$, *respectively. Also, given the storage function of ith process is induced by* $\Psi_i\geq 0\,\forall i\in[1,n_p]$. *Find a set of supply rates induced by* Φ, Φ_c *and* M_{com} *to satisfy the following LMI constraints*

$$\begin{pmatrix}\mathbb{T}_{11_i} & \mathbb{T}_{12_i} & \mathbb{T}_{13_i}\\ \mathbb{T}_{12_i}^T & \mathbb{T}_{22_i} & \mathbb{T}_{23_i}\\ \mathbb{T}_{13_i}^T & \mathbb{T}_{23_i}^T & \mathbb{T}_{33_i}\end{pmatrix}\geq 0\quad\forall i \tag{46a}$$

(Dissipativity of ith Process)

$$\tilde{\mathbb{X}}_{11}<0 \tag{46b}$$

(Stability of Plantwide Outputs)

$$\tilde{\mathbb{X}}_{11}\leq-\tilde{N}^T\tilde{N} \tag{46c}$$

$$\tilde{\mathbb{X}}_{13}=0 \tag{46d}$$

$$\tilde{\mathbb{X}}_{33}=\tilde{d}^T\tilde{d} \tag{46e}$$

(Guaranteed Plantwide Minimum Performance)

$$\Omega_i\geq 0\quad\forall i \tag{46f}$$

$$\chi_i<0\quad\forall i \tag{46g}$$

(Feasibility)
Conditions Eqs. 46c–46e guarantee the minimum plantwide performance level, as per Theorem 4 with the weighting function $\mathcal{W}=\frac{1}{d(\eta)}\tilde{N}(\eta)$.

If there is a solution to above offline problem, the proposed approach for the required plantwide performance and stability is feasible with the given communication rate. For a given indefinite $\Phi_c(\zeta,\eta)$, recursive feasibility may not be ensured, especially while applying hard input constraints. We recall the following sufficient condition to ensure the recursive feasibility in this case.

Proposition 3. (Tippett and Bao[25]). *For both cases with or without input constraints, a sufficient condition for the dissipative trajectory to be feasible is that the trajectory is originally dissipative and the constraint set* $u_c(k)\in\mathcal{U}$ *contains the origin and* $\Omega\geq 0$.

DMPC online algorithm

The required dissipativity conditions for individual controllers are enforced in the online MPC algorithm discussed below.

At $k=N\tilde{\tau}, N \in \mathbb{Z}^+$, ith controller receives predicted trajectories from other controllers, where the updated trajectory is used to determine control policy. Otherwise, the remote signal is processed via communication network as described in Eq. 26. At every time instant k, each controller solves the following optimization problem.

Problem 2. *(Online). Optimize the local control policy $\hat{u}_{c_i}$ by solving the following LMI problem*

$$\hat{u}_{c_i}(k)=\arg \min_{\alpha_i} \tag{47}$$

subject to

$$\begin{pmatrix} (\mathtt{diag}(\hat{Q}_i, \hat{R}_i))^{-1} & \begin{pmatrix} \hat{y}_{r_i} \\ \hat{u}_{c_i} \end{pmatrix} \\ \begin{pmatrix} \hat{y}_{r_i} \\ \hat{u}_{c_i} \end{pmatrix}^T & \alpha_i - w_y \epsilon_i \end{pmatrix} \geq 0 \tag{48a}$$

$$\begin{pmatrix} -\chi_i^{-1} & \hat{u}_{c_i} \\ \hat{u}_{c_i}^T & \Omega_i + W_{c_i}(k-\tilde{N}-1) \end{pmatrix} \geq 0 \tag{48b}$$

$$\hat{u}_{c_i} \in \mathcal{U} \tag{48c}$$

$$\hat{y}_{r_i} \in \mathcal{Y} \tag{48d}$$

$$\epsilon_i \geq 0 \tag{48e}$$

$$\hat{y}_{r_i}(k)=\check{A}_i \check{y}_i(k)+\check{B}_1 \check{u}_{c_i}(k)+\check{B}_2 \check{u}_{r_i}(k)+\hat{B}_1 \hat{u}_{c_i}(k)+\hat{B}_2 \hat{u}_{r_i}(k) \tag{48f}$$

where $\mathcal{U}$ and $\mathcal{Y}$ are any convex sets containing the origin and the matrices of $\hat{Q}$ and $\hat{R}$ are weighting functions with respects to $\hat{y}_r$ and $\hat{u}_c$ respectively.

At $k=N\tilde{\tau}$, $N \in \mathbb{Z}^+$, the ith controller sends its predicted trajectory $\hat{y}_r(k)$ of local process outputs to the other controllers via controller topology H_c, where the trajectory $\hat{y}_r(k)$ is the argument of the online problem while the control policy is optimized.

Remark 6. In this online problem, the conventional cost function in a quadratic manner is reformulated into an LMI condition as Eq. 48a (see details in Online problem section of Appendix A). Equation 48b is the condition to guarantee the recursive feasibility of the online problem. Equations 48a and 48b are input and output constraints in form of convex sets, where ϵ_i in Eq. 48c is used to penalize the local cost function when implementing soft constraints. The constraint as in Eq. 48d is the local process model to predict the future trajectories of local process outputs.

Illustrative Example

To illustrate the proposed approach, a case study of control of a network of a continuous stirred tank reactor (CSTR) and a distillation column (as shown in Figure 7) is presented in this section. While not a true large scale system, this network contains a recycling loop that leads to severe interactions between

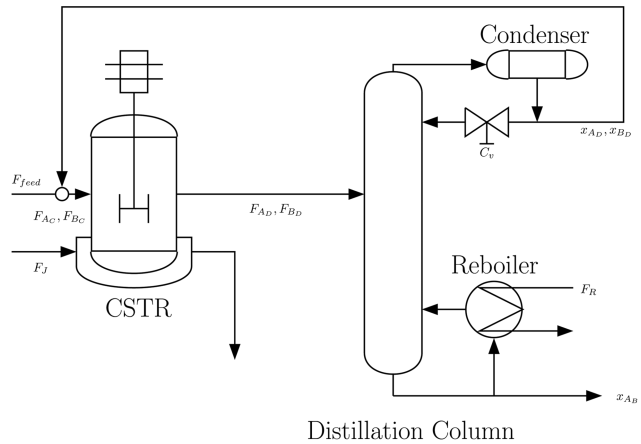


Figure 7. Plantwide system of CSTR and distillation column.

the distillation column and the reactor, and is useful to illustrate the effect of reduced-rate communication on a plantwide system. The models for the CSTR and distillation column are taken from Refs. 45 and 46, respectively, with the assumption of constant holdup volume, where the linearized discrete-time input-output models are given in models for illustrative example section in Appendix B. The irreversible reaction $A \rightarrow B$ occurs in the reactor. As per our network decomposition, the inputs to the reactor are partitioned as $u_{\mathrm{CSTR}}=\left(u_p^T|d^T|u_c^T\right)^T=\left(F_{A_C}^T, F_{B_C}^T|T_{J_{\mathrm{in}}}^T|F_J^T\right)^T$, where the separator only helps to distinguish variables. The u_p is the interconnecting inputs from distillation column and fresh feed F_{feed} are the molar flow rates of A and B with the disturbance consisting of a variation in the inlet cooling water temperature $T_{J_{\mathrm{in}}}$. The manipulated variable (local controller output, u_c) is the flow rate of water in the jacket, F_J. The inputs to the distillation column may be partitioned as $u_{\mathrm{DC}}=\left(u_p^T|d^T|u_c^T\right)^T= \left(F_{A_D}^T, F_{B_D}^T|T_s^T|F_R^T, C_v^T\right)^T$. That is, the interconnecting inputs recycling back to the CSTR, u_p, which are the molar flow rates of A and B (in the outflow of the reactor). The manipulated variables (local controller outputs), u_l, are the reflux flow rate (by manipulating a valve position, C_v) and the flow rate of the heat exchanger associated with reboiler F_R, respectively. The disturbance is reboiler temperature, which is denoted as T_s. The outputs are the molar flow rates of A and B in the distillation column, which is an interconnecting flow to the reactor, that is, $y_{\mathrm{DC}}=\left(x_{A_D}^T, x_{B_D}^T, x_{A_B}^T\right)^T$.

In all cases, three rectangular pulse disturbances are introduced: (1) a 2.5 K increase in CSTR jacket inlet temperature during $t=15-35$ h, (2) a 0.9 kg mol/h increase in the flow rate in reboiler heat exchanger during $t=40-60$ h, and (3) a fresh feed of 0.5 kg/h of reactant A into CSTR during $t=60-80$ h. The input of the CSTR and distillation column are constrained to be within $-1 \leq u_{l_1} \leq 1$ and $-0.5 \leq u_{l_2} \leq 0.5$, respectively.

To illustrate the effects on communication period, DMPC controllers without dissipativity-based plantwide stability and performance constraints are also simulated with reduced communication rate of $\tilde{\tau}=2$ (i.e., the communication period of 0.9 h, which is $(\tilde{\tau}+1)\times 0.3$ h), which is denoted as Case I. This DMPC optimizes the local cost function based on local measurements and the predicted trajectories from other

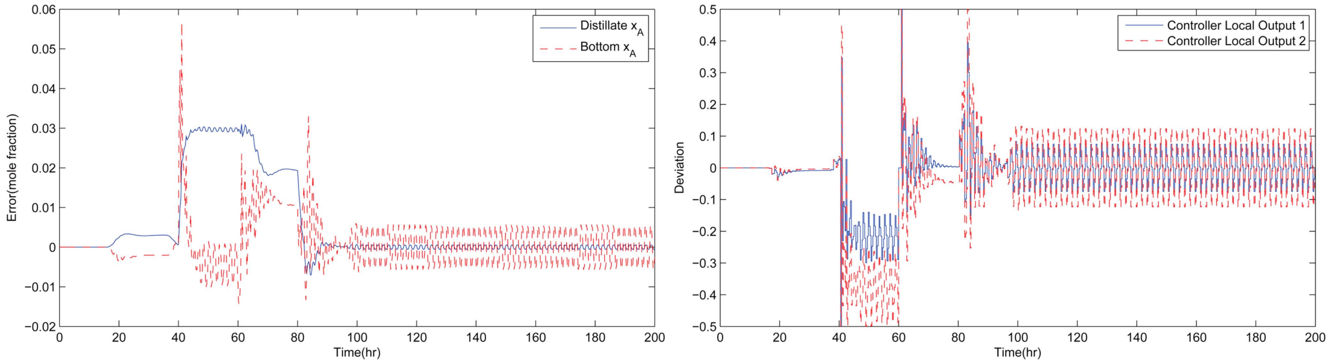


Figure 8. Selected controlled variables and controller outputs of the distillation column under DMPC without dissipativity conditions in Case I (communication period = 0.9 h).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

controllers. Furthermore, using the proposed approach, the increased communication periods $\tilde{\tau}$ are set as 1 and 2 (denoted as Case IIa and Case IIb, respectively) with the process topology H_p and controller topology H_c are off-diagonal matrices with appropriate dimension as given in Ref. 25, where

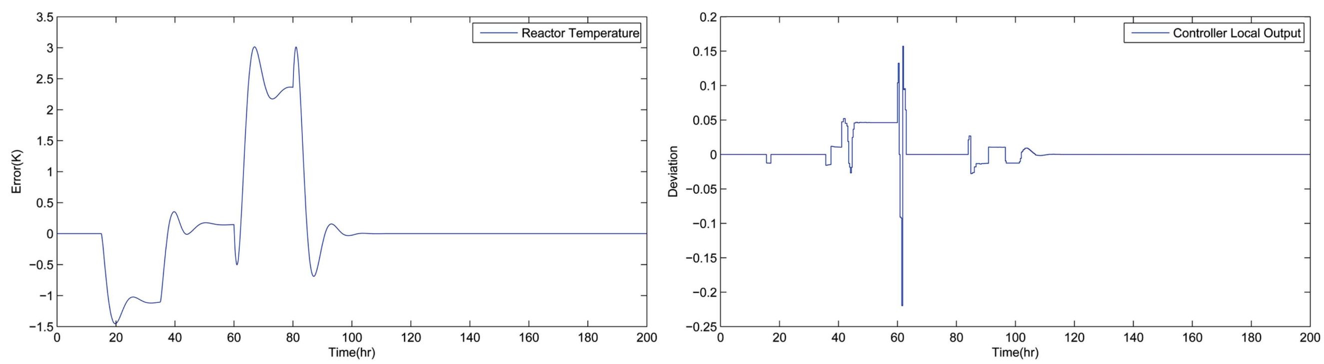
$$H_p = H_c = \begin{pmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & I_{3\times3} \\ I_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & I_{3\times3} & \mathbf{0}_{3\times3} \end{pmatrix} \tag{49}$$

The communication network $\hat{H}_c$ for cases of $\tilde{\tau}=1$ and $\tilde{\tau}=2$ are given as below, respectively

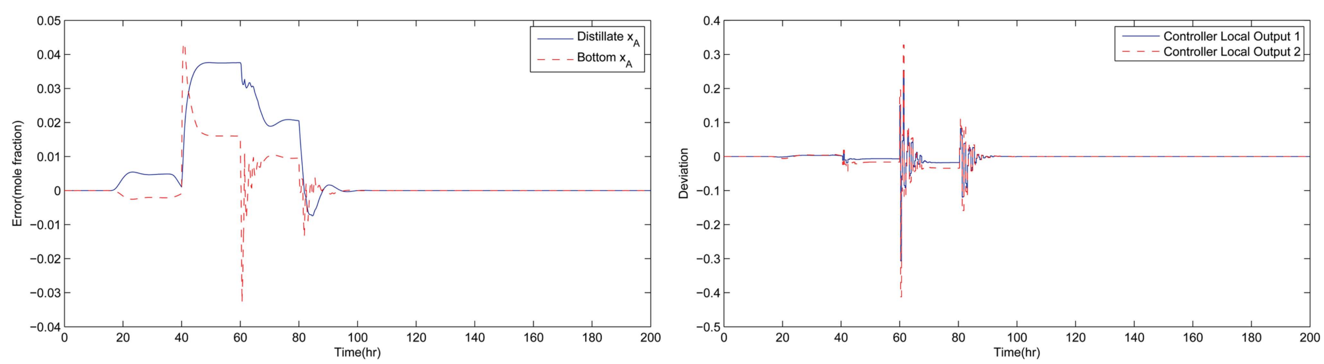
$$\hat{H}_c = \begin{pmatrix} H_c & \mathbf{0} \\ H_c & \mathbf{0} \end{pmatrix} \quad \text{and} \quad \hat{H}_c = \begin{pmatrix} H_c & \mathbf{0} & \mathbf{0} \\ H_c & \mathbf{0} & \mathbf{0} \\ H_c & \mathbf{0} & \mathbf{0} \end{pmatrix} \tag{50}$$

While the assumption of constant holdup volume may eliminate some of the fast dynamics of the system, the dynamics of the distillation column still has a bandwidth of 1 rad/h, which warrants the use of a sampling period of 0.3 h. This allows the proposed approach to implement a communication rate as slow as once every 0.9 h without causing plantwide instability and significantly reduced performance.

Figure 8 shows the plantwide system in Case I is unstable with the controlled variables and controller output becoming



(a) CSTR



(b) Distillation Column

Figure 9. Selected controlled variable and controller outputs in Case IIa (communication period = 0.6 h).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

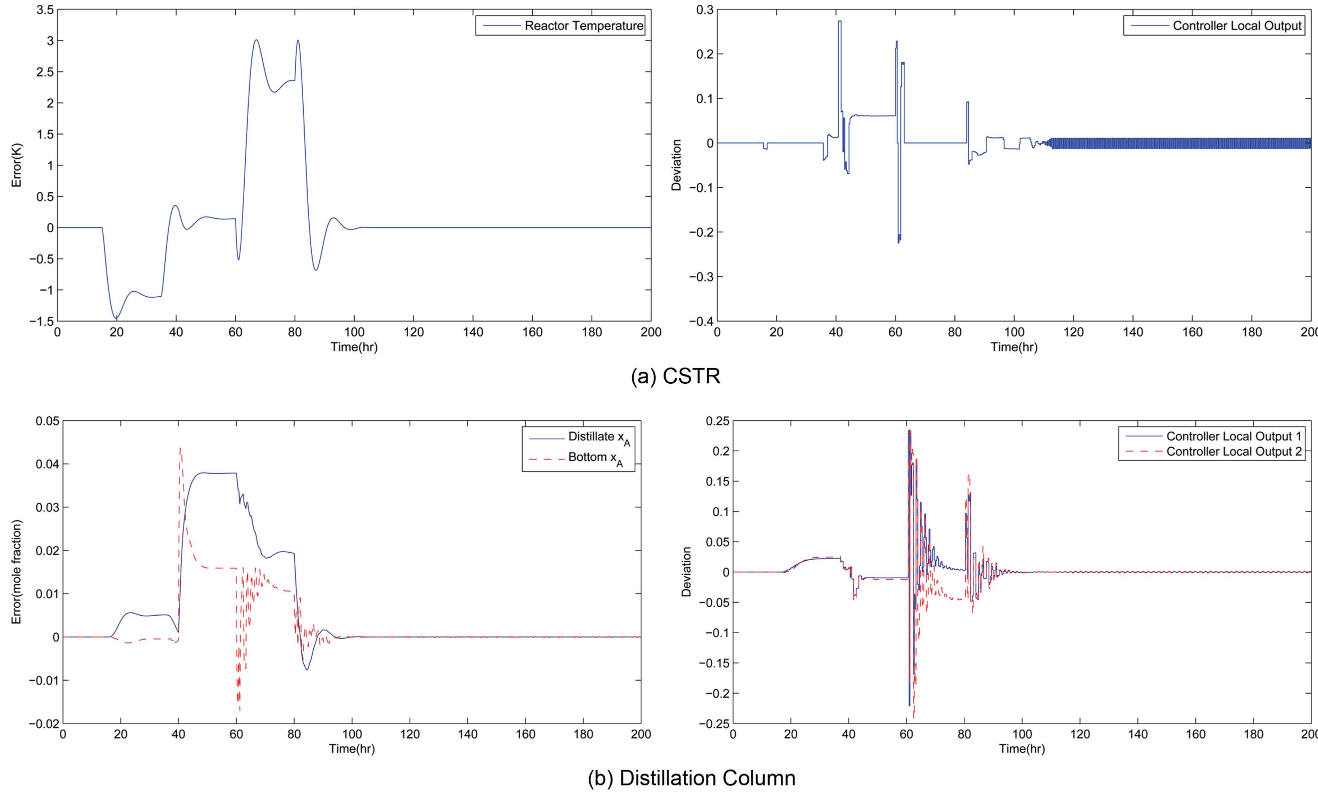


Figure 10. Selected controlled variable and controller outputs in Case IIb (communication period = 0.9 h).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

oscillatory with less frequent communication. By applying the dissipativity-based approach developed in this article (Case II), the plantwide system is stabilized with longer communication periods of 0.6 h ($\tilde{\tau}=1$) and 0.9 h ($\tilde{\tau}=2$), as shown in Figures 9 and 10, respectively. It is clear that the controlled variables converge after 100 h, where the integral of absolute errors of all controlled variables in the plantwide system are 7.3407 and 7.3418 for the case of 0.6 h and 0.9 h, respectively. It is evident that the control performance maintains at the same level with using reduced-rate communication. Furthermore, the integral of absolute values of the controller outputs are 2.6611 and 5.2519 for these two cases, respectively, showing that a slower communication rate does lead to less efficient control action. While ensuring plantwide stability, the performance of dissipativity-based DMPC decreases with a longer communication period. This is expected because the dynamic communication error gain becomes larger (as discussed in Remark 3).

Conclusions

This article investigates the effects of reduced communication rates on the stability and performance of DMPC. This analysis is developed using the lifting technique and the concept of dissipativity. This is facilitated by the use of QdFs as dynamic supply rates and storage functions, which provide a convenient framework for the dissipativity-based analysis of the plantwide system both before and after lifting. The dissipative trajectory conditions, which each controller needs to satisfy to ensure plantwide stability and performance, are developed in the lifted sampling rate. The proposed approach is shown effective by an illustrative case study of DMPC with different communication rates.

It is possible to extend the results in this article to nonlinear systems. The main challenge is to determine the dynamic supply rates (in QdF) for nonlinear systems. This is possible by extending our recent work on plantwide nonlinear control based on (Q, S, R)-dissipativity, that is, Refs. 47 and 48. This is possible because the QdF dissipativity of a system is equivalent to the QSR-dissipativity of the same system augmented with the extended input and output.[49] The problem of determining the QdF dissipativity of a nonlinear system is equivalent to finding the QSR-dissipativity of the augmented system. Another possible future work is to investigate the optimal topologies of communication networks (with different communication rates), which are trade-offs between the complexity of networks and plantwide control performance.

Acknowledgment

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Appendix A: Proofs

Theorem 4

Proof. According to Proposition 1, the plantwide system traces a dissipative trajectory for all time instants k, which implies

$$\sum_{t=0}^{k} \tilde{\hat{y}}_{\mathrm{pw}}^{T}(t)\tilde{\mathbb{X}}_{11}\tilde{\hat{y}}_{\mathrm{pw}}(t)+2\tilde{\hat{y}}_{\mathrm{pw}}^{T}(t)\tilde{\mathbb{X}}_{13}\tilde{\hat{d}}(t)+\tilde{\hat{d}}^{T}(t)\tilde{\mathbb{X}}_{33}\tilde{\hat{d}}(t) \geq Q_{\Psi}(k+1)-Q_{\Psi}(0) \tag{A1}$$

where the order of extended signals is equal to $\tilde{n}\tilde{\tau}$. For convenience, the time dependence is dropped in the following inequalities. By assuming, the system has the condition $Q_{\Psi}(k+1)=Q_{\Psi}(0)=0$. we have

$$\sum_{t=0}^{k} \tilde{\hat{d}}^{T}\tilde{\mathbb{X}}_{33}\tilde{\hat{d}} \geq \sum_{t=0}^{k} \tilde{\hat{y}}_{\mathrm{pw}}^{T}\hat{\tilde{\mathbb{X}}}_{11}\tilde{\hat{y}}_{\mathrm{pw}}-2\tilde{\hat{y}}_{\mathrm{pw}}^{T}\tilde{\mathbb{X}}_{13}\tilde{\hat{d}} \tag{A2}$$

where $\hat{\tilde{\mathbb{X}}}_{11}=-\tilde{\mathbb{X}}_{11}$. Completing the square on right-hand side leads to

$$\sum_{t=0}^{k} [\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}}\tilde{\hat{y}}_{\mathrm{pw}}-\hat{\tilde{\mathbb{X}}}_{11}^{-\frac{1}{2}}\tilde{\mathbb{X}}_{13}]^{T}[\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}}\tilde{\hat{y}}_{\mathrm{pw}}-\hat{\tilde{\mathbb{X}}}_{11}^{-\frac{1}{2}}\tilde{\mathbb{X}}_{13}] \leq \sum_{t=0}^{k} \tilde{\hat{d}}^{T}[\tilde{\mathbb{X}}_{33}+\tilde{\mathbb{X}}_{13}^{T}\hat{\tilde{\mathbb{X}}}_{11}\tilde{\mathbb{X}}_{13}^{T}]\tilde{\hat{d}} \tag{A3}$$

Let p be a row vector with appropriate dimension such that $p^T p \geq \max(\tilde{\mathbb{X}}_{33}+\tilde{\mathbb{X}}_{13}^T\hat{\tilde{\mathbb{X}}}_{11}\tilde{\mathbb{X}}_{13}^T, \tilde{\mathbb{X}}_{13}^T\hat{\tilde{\mathbb{X}}}_{11}\tilde{\mathbb{X}}_{13}^T)$. The following inequality can be obtain using reverse triangle inequality

$$||\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}}\tilde{\hat{y}}_{\text{pw}}||-||\hat{\tilde{\mathbb{X}}}_{11}^{-\frac{1}{2}}\tilde{\mathbb{X}}_{13}\hat{d}|| \leq ||p\tilde{\hat{d}}|| \tag{A4}$$

$$||\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}}\tilde{\hat{y}}_{\text{pw}}|| \leq ||\hat{\tilde{\mathbb{X}}}_{11}^{-\frac{1}{2}}\tilde{\mathbb{X}}_{13}\hat{d}||+||p\tilde{\hat{d}}|| \tag{A5}$$

$$||\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}}\tilde{\hat{y}}_{\text{pw}}|| \leq 2||p\tilde{\hat{d}}|| \tag{A6}$$

It is clear that $||\tilde{\hat{y}}_{\text{pw}}|| \leq \gamma||\tilde{\hat{d}}||$, if

$$\frac{\hat{\tilde{\mathbb{X}}}_{11}^{\frac{1}{2}}}{2p} \geq \frac{1}{\gamma} \tag{A7}$$

■

Proposition 3

The supply rate of ith controller, $Q_{\phi_{c_i}}$, is in term of $\hat{w}_{c_i}=\left(\hat{y}_l^T,\hat{y}_r^T,\hat{u}_l^T,\hat{u}_r^T\right)^T$. The controller traces s nonnegative dissipative trajectory, which is

$$\mathcal{W}_{c_i}(k-\tilde{N}-1)=\sum_{k=0}^{\tilde{N}-1} Q_{\phi_{c_i}}+\hat{w}_{c_i}^T Q_{\phi_{c_i}}\hat{w}_{c_i} \geq 0 \tag{A8}$$

By applying the process model and taking Schur component, this inequality can be rearranged to as an LMI condition, which is

$$\begin{pmatrix} -\chi_i^{-1} & \hat{u}_{c_i} \\ \hat{u}_{c_i} & \Omega_i+\mathcal{W}_{c_i}(k-\tilde{N}-1) \end{pmatrix} \geq 0 \tag{A9}$$

where

$$\Omega_i=\begin{pmatrix} \Omega_{11_i} & \Omega_{12_i} \\ \Omega_{12_i}^T & \Omega_{22_i} \end{pmatrix} \tag{A10a}$$

$$\chi_i=(Q_{lr_i}+S_{ll_i})\hat{B}_{2_i}+\hat{B}_{2_i}^T(Q_{lr_i}+S_{ll_i})^T+\hat{B}_{2_i}^T\mathbb{F}\hat{B}_{2_i} \tag{A10b}$$

$$\Omega_{11_i}=\begin{pmatrix} \mathbf{0} & (Q_{lr_i}+S_{ll_i})\hat{B}_{1_i}+S_{lr_i}+\hat{B}_{2_i}^T\mathbb{F}\hat{B}_{1_i}+\hat{B}_{2_i}R_{lr_i} \\ * & \hat{B}_{1_i}^T\mathbb{F}\hat{B}_{1_i}+\hat{B}_{1_i}^T(S_{rr_i}^T+R_{lr_i})+(S_{rr_i}^T+R_{lr_i})^T\hat{B}_{1_i}+R_{rr_i} \end{pmatrix} \tag{A10c}$$

$$\Omega_{12_i}=\begin{pmatrix} (Q_{lr_i}+S_{ll_i}+\hat{B}_{2_i}^T\mathbb{F})\check{A}_i, & (Q_{lr_i}+S_{ll_i}+\hat{B}_{2_i}^T\mathbb{F})\check{B}_{2_i}, & (Q_{lr_i}+S_{ll_i}+\hat{B}_{2_i}^T\mathbb{F})\check{B}_{1_i} \\ (\check{B}_{2_i}^T\mathbb{F}+S_{rr_i}^T+R_{lr_i})\check{A}_i, & (\check{B}_{2_i}^T\mathbb{F}+S_{rr_i}^T+R_{lr_i})\check{B}_{2_i}, & (\check{B}_{2_i}^T\mathbb{F}+S_{rr_i}^T+R_{lr_i})\check{B}_{1_i} \end{pmatrix} \tag{A10d}$$

$$\Omega_{22_i}=\begin{pmatrix} \check{A}_i^T\mathbb{F}\check{A}_i & \check{A}_i^T\mathbb{F}\check{B}_{2_i} & \check{A}_i^T\mathbb{F}\check{B}_{1_i} \\ * & \check{B}_{2_i}^T\mathbb{F}\check{B}_{2_i} & \check{B}_{2_i}^T\mathbb{F}\check{B}_{1_i} \\ * & * & \check{B}_{1_i}^T\mathbb{F}\check{B}_{1_i} \end{pmatrix} \tag{A10e}$$

$$\mathbb{F}=Q_{rr_i}+S_{rl_i}+S_{rl_i}^T+R_{ll_i} \tag{A10f}$$

Online problem

The cost function of ith controller J_i is given as

$$J_i=\begin{pmatrix} \hat{y}_{r_i} \\ \hat{u}_{c_i} \end{pmatrix}^T \begin{pmatrix} \hat{Q}_i & 0 \\ 0 & \hat{R}_i \end{pmatrix} \begin{pmatrix} \hat{y}_{r_i} \\ \hat{u}_{c_i} \end{pmatrix}+w_y\epsilon \tag{A11}$$

where $\hat{Q}$ and $\hat{R}$ are diagonal weighting functions with respect to $\hat{y}_r$ and $\hat{u}_c$, respectively, and $w_y\epsilon$ is a penalty function for applying soft constraint. The minimization of this cost function is equivalent to the minimization of α in the following inequality

$$\alpha_i-J_i \geq 0 \tag{A12}$$

By taking Schur complement, this inequality becomes

$$\begin{pmatrix} (\text{diag}(\hat{Q}_i,\hat{R}_i))^{-1} & \begin{pmatrix} \hat{y}_{r_i} \\ \hat{u}_{c_i} \end{pmatrix} \\ \begin{pmatrix} \hat{y}_{r_i} \\ \hat{u}_{c_i} \end{pmatrix}^T & \alpha_i-w_y\epsilon_i \end{pmatrix} \geq 0 \tag{A13}$$

subject to the process model constraints.

Appendix B: Illustrative Example

Models for illustrative example

The discrete-time input-output model for the CSTR at a sampling rate of 0.3 h is

$$y_c(k)=\sum_{i=1}^{4} A_{c_i}y_c(k-i)+\sum_{j=0}^{4} B_{c_j}u(k-j) \tag{B1}$$

where

$$A_{c_1}=2.0932I_{3\times3} \tag{B2a}$$

$$A_{c_2}=-1.615I_{3\times3} \tag{B2b}$$

$$A_{c_3}=0.4584I_{3\times3} \tag{B2c}$$

$$A_{c_4}=-0.002311I_{3\times3} \tag{B2d}$$

$$B_{c_0}=\begin{pmatrix} 0.34 & 0 & 0 & 0 \\ 0.85 & 0.2517 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{B2e}$$

$$B_{c_1}=\begin{pmatrix} -0.5616 & 0 & 0.001724 & 0.001724 \\ -1.976 & 0.3385 & 0.001719 & 0.001719 \\ -1.30202 & 0 & -0.1615 & 0.1615 \end{pmatrix} \tag{B2f}$$

$$B_{c_2}=\begin{pmatrix} 0.3013 & 0 & 0.002602 & 0.002602 \\ 1.684 & 0.0172 & 0.001288 & 0.001288 \\ 3.4573 & 0 & -0.1234 & 0.1234 \end{pmatrix} \tag{B2g}$$

$$B_{c_3}=\begin{pmatrix} -0.0530 & 0 & -0.0018 & -0.0018 \\ -0.5213 & 0.0008 & 0.001833 & 0.001833 \\ -1.8722 & 0 & 0.0169 & 0.0169 \end{pmatrix} \tag{B2h}$$

$$B_{c_4}=\begin{pmatrix}-0.0003 & 0 & -0.0001 & -0.0001\\ 0.002625 & 0 & 9.221\times10^{-5} & 9.221\times10^{-5}\\ 0.0106 & 0 & -0.0141 & -0.0141\end{pmatrix} \tag{B2i}$$

The discrete input-output model of the distillation column at a sampling rate of 0.3 h is

$$y_d(k)=\sum_{i=1}^{7}A_{d_i}y_c(k-i)+\sum_{j=0}^{7}B_{d_j}u(k-j) \tag{B3}$$

where

$$A_{d_1}=0.8981I_{3\times3} \tag{B4a}$$

$$A_{d_2}=-0.2165I_{3\times3} \tag{B4b}$$

$$A_{d_3}=0.01932I_{3\times3} \tag{B4c}$$

$$A_{d_4}=-0.0009418I_{3\times3} \tag{B4d}$$

$$A_{d_5}=4.563\times10^{-5}I_{3\times3} \tag{B4e}$$

$$A_{d_6}=-7.058\times10^{-7}I_{3\times3} \tag{B4f}$$

$$A_{d_7}=3.005\times10^{-9}I_{3\times3} \tag{B4g}$$

$$B_{d_0}=\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix} \tag{B4h}$$

$$B_{d_1}=\begin{pmatrix}0.04142 & 0.0107 & 0.0198 & -0.01362 & 0.0198\\ -0.02022 & -0.02022 & -0.04788 & -0.03645 & -0.04788\\ 0.02022 & 0.02688 & 0.04788 & 0.03645 & 0.02688\end{pmatrix} \tag{B4i}$$

$$B_{d_2}=\begin{pmatrix}0.003948 & -0.01527 & -0.007094 & 0.001609 & 0.007094\\ 0.02804 & 0.01579 & 0.05351 & 0.01908 & 0.05351\\ -0.02804 & -0.01579 & -0.05351 & -0.01908 & -0.05351\end{pmatrix} \tag{B4j}$$

$$B_{d_3}=\begin{pmatrix}-0.01392 & 0.00389 & 0.0005736 & 0.002733 & -0.0005736\\ -0.003353 & -0.003504 & -0.01206 & -0.03126 & -0.01206\\ 0.003353 & 0.003504 & 0.01206 & 0.03126 & 0.01206\end{pmatrix} \tag{B4k}$$

$$B_{d_4}=\begin{pmatrix}0.0001844 & 0.0001342 & -9.42\times10^{-6} & -0.0004859 & -9.42\times10^{-6}\\ 0.0001087 & 0.0002813 & 0.0009005 & 0.0001936 & 0.0009005\\ -0.0001087 & -0.0002813 & -0.0009005 & -0.0001936 & -0.0009005\end{pmatrix} \tag{B4l}$$

$$B_{d_5}=\begin{pmatrix}9.271\times10^{-5} & -3.68\times10^{-5} & -9.977\times10^{-6} & 5.6838\times10^{-6} & -9.977\times10^{-6}\\ -1.407\times10^{-5} & -1.008\times10^{-5} & -4.4115\times10^{-5} & -1.007\times10^{-5} & -4.4115\times10^{-5}\\ 1.407\times10^{-5} & 1.008\times10^{-5} & 4.115\times10^{-5} & 1.007\times10^{-5} & 4.115\times10^{-5}\end{pmatrix} \tag{B4m}$$

$$B_{d_6}=\begin{pmatrix}-5.192\times10^{-7} & 1.461\times10^{-7} & -4.608\times10^{-8} & 6.18\times10^{-8} & -4.608\times10^{-8}\\ 2.28\times10^{-7} & -5.527\times10^{-7} & 1.884\times10^{-6} & 2.944\times10^{-7} & 1.884\times10^{-6}\\ -2.28\times10^{-7} & 5.527\times10^{-7} & -1.884\times10^{-6} & -2.944\times10^{-7} & -1.884\times10^{-6}\end{pmatrix} \tag{B4n}$$

$$B_{d_7}=\begin{pmatrix}-8.47\times10^{-9} & 3.049\times10^{-9} & 7.194\times10^{-11} & -9.694\times10^{-10} & 7.194\times10^{-11}\\ -4.292\times10^{-9} & -2.487\times10^{-9} & -1.196\times10^{-8} & -1.832\times10^{-9} & -1.196\times10^{-8}\\ 4.292\times10^{-9} & 2.487\times10^{-9} & 1.196\times10^{-8} & 1.832\times10^{-9} & 1.196\times10^{-8}\end{pmatrix} \tag{B4o}$$

Dissipativity properties for illustrative example

The dissipativity of the lifted closed-loop system can be determined by solving offline problem. The below is an indicative example of the form of the supply rates for the switched communication network $\hat{H}_c$ with $\tilde{\tau}=2$. In the QdFs, the supply rate of such communication network is induced by

$$\begin{pmatrix} \tilde{\mathcal{Q}}_{\mathrm{com}} & \tilde{\mathcal{S}}_{\mathrm{com}} \\ \tilde{\mathcal{S}}_{\mathrm{com}}^{T} & \tilde{\mathcal{R}}_{\mathrm{com}} \end{pmatrix} \tag{B5}$$

where

$$\tilde{\mathcal{Q}}_{\mathrm{com}}=\begin{pmatrix}
2.4845 & -0.0013 & -0.0099 & 0.0023 & 0 & 0 & 0 & 0 & -2.4844 & 0.0013 & 0.0098 & -0.0023 \\
-0.0013 & 2.4793 & 0.0043 & -0.0106 & 0 & 0.0001 & 0 & 0 & 0.0013 & -2.4793 & -0.0043 & 0.0106 \\
-0.0099 & 0.0043 & 2.4872 & 0.0109 & 0 & 0 & -0.0001 & 0 & 0.0099 & -0.0043 & -2.4872 & -0.0108 \\
0.0023 & -0.0106 & 0.0109 & 2.4928 & 0 & 0 & 0 & 0 & -0.0023 & 0.0106 & -0.0108 & -2.4927 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.0001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.0001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2.4844 & 0.0013 & 0.0099 & -0.0023 & 0 & 0 & 0 & 0 & 2.4844 & -0.0014 & -0.0099 & 0.0023 \\
0.0013 & -2.4793 & -0.0043 & 0.0106 & 0 & 0 & 0 & 0 & -0.0014 & 2.4793 & 0.0043 & -0.0106 \\
0.0098 & -0.0043 & -2.4872 & -0.0108 & 0 & 0 & 0.0001 & 0 & -0.0099 & 0.0043 & 2.4871 & 0.0108 \\
-0.0023 & 0.0106 & -0.0108 & -2.4927 & 0 & 0 & 0 & 0 & 0.0023 & -0.0106 & 0.0108 & 2.4927
\end{pmatrix} \tag{B6a}$$

$$\tilde{\mathcal{S}}_{\mathrm{com}}=10^{-4}\times\begin{pmatrix}
-0.2588 & -0.1254 & -0.3194 & -0.1495 & -0.0032 & -0.0001 & -0.0008 & 0 & 0.2580 & 0.1253 & 0.3169 & 0.1494 \\
-0.1254 & -0.2670 & -0.2015 & -0.4446 & -0.0001 & -0.0032 & -0.0001 & -0.0008 & 0.1253 & 0.2662 & 0.2014 & 0.4421 \\
-0.3194 & -0.2015 & 0.7910 & -0.2867 & -0.0023 & 0 & -0.0026 & 0 & 0.3176 & 0.2015 & -0.7924 & 0.2867 \\
-0.1495 & -0.4446 & -0.2867 & 0.2893 & -0.0001 & -0.0023 & -0.0001 & -0.0026 & 0.1495 & 0.4428 & 0.2866 & -0.2907 \\
-0.0032 & -0.0001 & -0.0023 & -0.0001 & -0.0010 & 0 & -0.0017 & -0.0001 & 0.0005 & 0.0001 & 0.0005 & 0 \\
-0.0001 & -0.0032 & 0 & -0.0023 & 0 & -0.0010 & -0.0001 & -0.0018 & 0.0001 & 0.0005 & 0 & 0.0006 \\
-0.0008 & -0.0001 & -0.0026 & -0.0001 & -0.0017 & -0.0001 & -0.0009 & 0 & 0.0003 & 0 & 0.0003 & 0 \\
0 & -0.0008 & 0 & -0.0026 & -0.0001 & -0.0018 & 0 & -0.0009 & 0 & 0.0003 & 0 & 0.0002 \\
0.2580 & 0.1253 & 0.3176 & 0.1495 & 0.0005 & 0.0001 & 0.0003 & 0 & -0.2578 & -0.1253 & -0.3171 & -0.1494 \\
0.1253 & 0.2662 & 0.2015 & 0.4428 & 0.0001 & 0.0005 & 0 & 0.0003 & -0.1253 & -0.2661 & -0.2014 & -0.4423 \\
0.3169 & 0.2014 & -0.7924 & 0.2866 & 0.0005 & 0 & 0.0003 & 0 & -0.3171 & -0.2014 & 0.7926 & -0.2866 \\
0.1494 & 0.4421 & 0.2867 & -0.2907 & 0 & 0.0006 & 0 & 0.0002 & -0.1494 & -0.4423 & -0.2866 & 0.2909
\end{pmatrix} \tag{B6b}$$

$$\tilde{\mathcal{R}}_{\mathrm{com}} = 10^{-6} \times \begin{pmatrix} 0.2822 & 0.0067 & 0.1458 & 0.0068 & 0.0890 & 0.0051 & 0.1686 & 0.0053 & -0.0291 & -0.0051 & -0.0325 & -0.0022 \\ 0.0067 & 0.2871 & 0.0075 & 0.1469 & 0.0011 & 0.1021 & 0.0037 & 0.1744 & -0.0052 & -0.0299 & -0.0014 & -0.0348 \\ 0.1458 & 0.0075 & 0.3116 & 0.0107 & 0.1789 & 0.0069 & 0.1126 & 0.0095 & -0.0149 & -0.0009 & -0.0342 & 0.0010 \\ 0.0068 & 0.1469 & 0.0107 & 0.2987 & 0.0043 & 0.1737 & 0.0029 & 0.1183 & -0.0039 & -0.0200 & -0.0015 & -0.0346 \\ 0.0890 & 0.0011 & 0.1789 & 0.0043 & 0.3183 & -0.0030 & 0.0704 & 0.0039 & -0.0056 & -0.0010 & -0.0058 & -0.0032 \\ 0.0051 & 0.1021 & 0.0069 & 0.1737 & -0.0030 & 0.2997 & 0.0078 & 0.0741 & -0.0010 & -0.0089 & 0.0043 & -0.0019 \\ 0.1686 & 0.0037 & 0.1126 & 0.0029 & 0.0704 & 0.0078 & 0.1042 & 0.0080 & -0.0236 & -0.0046 & -0.0166 & -0.0001 \\ 0.0053 & 0.1744 & 0.0095 & 0.1183 & 0.0039 & 0.0741 & 0.0080 & 0.1133 & 0.0010 & -0.0198 & 0.0006 & -0.0137 \\ -0.0291 & -0.0052 & -0.0149 & -0.0039 & -0.0056 & -0.0010 & -0.0236 & 0.0010 & -0.0291 & 0.0097 & 0.0121 & 0.0041 \\ -0.0051 & -0.0299 & -0.0009 & -0.0200 & -0.0010 & -0.0089 & -0.0046 & -0.0198 & 0.0097 & -0.0224 & 0.0081 & 0.0131 \\ -0.0325 & -0.0014 & -0.0342 & -0.0015 & -0.0058 & 0.0043 & -0.0166 & 0.0006 & 0.0121 & 0.0081 & -0.0110 & 0.0090 \\ -0.0022 & -0.0348 & 0.0010 & -0.0346 & -0.0032 & -0.0019 & -0.0001 & -0.0137 & 0.0041 & 0.0131 & 0.0090 & -0.0127 \end{pmatrix} \tag{B6c}$$

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